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LETTER TO THE EDITOR

Blackish fractal balls†

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Abstract. The blackish area M_u of crumpled paper balls of dimension $D=2.5$ obtained through the immersion of such objects in a black solution or with the aid of a ballistic painting technique is investigated. Among other scaling relations we find that in the first case $M_u \sim L^{1.39\pm0.16}$ and in the second case $M_u \sim L^{1.24\pm0.12}$ where L is the unfolded size.

An extensive statistical analysis has shown that the apparently ill-defined procedure of formation of crumpled paper balls generates mass fractals which are random surfaces satisfying the relation $M = kR^D$ between mass M and size R, with $D=2.5$ and relative uncertainties $(\Delta D/D)=0.07$ and $(\Delta k/k)=0.12$ within a large interval of sizes and surface densities [1].

In the present work the painted area M_u of such surfaces immersed in a water-soluble black pigment is investigated. Three groups of crumpled paper balls with unfolded square areas varying from $M=1$, corresponding to a square of edge $L=13.5$ mm, to $M=256$ ($L=216$ mm) and immersion times of 35 min, 70 min and 130 min were tested. The paper used had a density of 75.8 g m^{-2} and a high capacity to fix the pigment in the region of contact with the solution. In these circumstances the diffusion of the pigment through the surface is minimised and the obtained painted area is a direct measure of the unscreened perimeter of the fractal. The ensemble of surfaces in contact with the solution during 35 min was formed by five equal sets of balls with unfolded sizes $L=13.5$, 27, 54, 108 and 216 mm. Each one of the two other groups (immersion times of 70 and 130 min) was formed by five equal sets of balls with unfolded sizes $L=27$, 54, 108 and 216 mm. Thus we have examined altogether 65 two-side surfaces. There are three main conclusions of this study. (i) The number $N_u(\varepsilon)$ of bidimensional squares of edge ε, needed to cover the painted area or unscreened perimeter $M_u \equiv \varepsilon^2 N_u(\varepsilon)$ of each ball, exhibits after a perfect unfolding the scale dependence $N_u(\varepsilon) \sim \varepsilon^{-D_1}$, with $D_1 = 1.34 \pm 0.15$ throughout the scale interval $2\text{ mm} \leqslant \varepsilon \leqslant L/3$ mm utilised. This exponent D_1 measured is independent of L in all intervals of sizes investigated. (ii) When painted areas M_u associated with different L are compared using the same scale ε, it is found that $M_u \sim L^{D_2}$, with $D_2 = 1.39 \pm 0.16$ within the interval of L studied. This exponent D_2 is independent of ε for the interval $2\text{ mm} \leqslant \varepsilon \leqslant L/3$ mm. (iii) Furthermore, all these results are independent of the time of immersion. Additionally these numbers do not change if we consider the painted area on a single face of the sheet or as the sum of the painted area on the two faces. More detailed information concerning D_1 and D_2 is given in table 1.

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Table 1. The values of the exponents D_1 and D_2 defined in the text and the respective uncertainties for three different times T of immersion.

	$T=35$ min	$T=70$ min	$T=130$ min
D_1	1.38 ± 0.15	1.31 ± 0.27	1.31 ± 0.19
D_2	1.32 ± 0.19	1.48 ± 0.23	1.42 ± 0.22

We interpret the black regions of the unfolded sheets (figure 1) as fractal sets of dimension $D_1 \simeq D_2$. As a consequence of the dependence of M_u with L, the fraction F of painted area with respect to the total unfolded area $M \equiv L^2$ behaves as $F \sim L^{D_2-2} = L^{-0.61}$, and consequently goes to zero when L increases. This fact can be observed in figure 1 which reproduces a family of four unfolded balls with plane areas varying by a factor of 64. Since the average radius R of each ball scales with L as $R \sim L^{2/D}$, we have $M_u \sim L^{D_2} \sim R^{d_c}$, with $d_c = D_2D/2 = 5D_2/4$, for $D=5/2$ as previously observed [1]. Thus using the experimental value of D_2 we find that the crumpled painted area has a fractal dimension of $d_c = 1.74\pm0.2$ in the tridimensional space. We note in passing that this exponent d_c is in numerical agreement with the exponent $d_u = (D-1)+(d-D)/d_p$ found by Coniglio and Stanley [2] for the unscreened perimeter of an arbitrary fractal. In the last expression D is the dimension of an arbitrary fractal embedded in a space of dimension d and d_p is the fractal dimension for the trajectory of the projectiles. For the kind of fractal surface considered in this letter the corresponding d_u is 1.75, since $D=2.5$, $d=3$ and the pigment particles are assumed to perform random walks whose fractal dimension of the trajectory is $d_p = 2$.

In order to study the role of other fractal dimensions of the pigment-particle trajectories upon M_u a second experiment was realised using the same black pigment applied with a low-flux air brush keeping an outlet pressure of 2 atm and a dispersion angle of 5°. Twenty-four crumpled balls of sizes $L=27$, 54, 108 and 216 mm were painted in this case, the black areas now presenting very sharp boundaries (figure 2). The measured exponents are $D_1 = 1.21\pm0.15$ and $D_2 = 1.24\pm0.12$. Thus $d_c = 5D_2/4 = 1.55\pm0.15$, which is not in concordance with the corresponding value $d_u = 2$ obtained with equation (2) of [2] when $d_f \equiv D = 2.5$, $d_p = 1$ and $d=3$. This discrepancy must be seen as a consequence of experimental difficulties and not as a disproof of the result reported in [2]. Firstly it is not easy to avoid interactions of the air-brush jet with the air. These interactions tend to destroy the linearity of the pigment trajectories and this may lead to a change from an initial regime of $d_p = 1$ to a regime of $d_p \simeq 2$. On the other hand, if the air-brush outlet pressure is increased beyond 2 atm in order to obtain straight pigment trajectories of great reliability then we obtain as a consequence severe distortions of the fractal surfaces. Another possibility which we cannot neglect in order to explain the discrepancy between the values of D_2 is connected with the observation that the flow of pigment droplets near the random surface is complex and certainly turbulent in a high degree. This may be a result of the irregularities and discontinuities in the surface of the fractal balls. Curiously if we assume that $D_2 = 1.24$ is compatible with the expression for d_u given in [2], i.e. $(5/4)D_2 = 1.55 = (D-1)+(d-D)/d_p$, we find that $2.55 \leqslant d \leqslant 2.6$ for $1 \leqslant d_p \leqslant 2$ and $D=2.5$. That is, in this kind of painting the random surface could be imagined as primarily or effectively embedded in a fractal set of dimension d around 2.6 (the last set in its turn is embedded in the tridimensional space). It is known that the region where turbulent activity is concentrated is seemingly a fractal of dimension d in the interval $2.5 \leqslant d \leqslant 2.7$ [3, 4].

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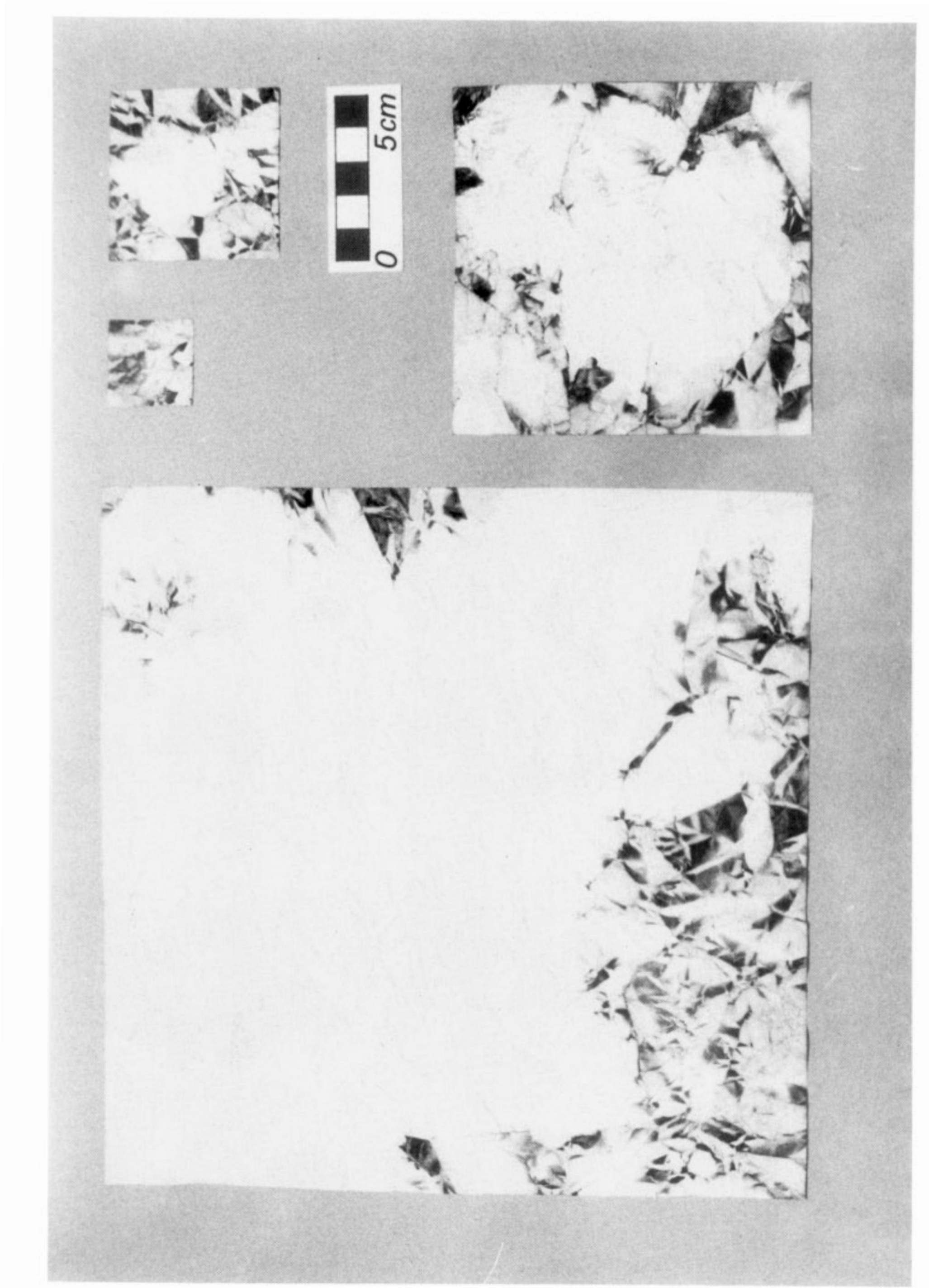

Figure 1. Unfolded balls obtained after immersion in a black pigment solution, $T = 130$ min (see text and table 1).

[facing p L1168]

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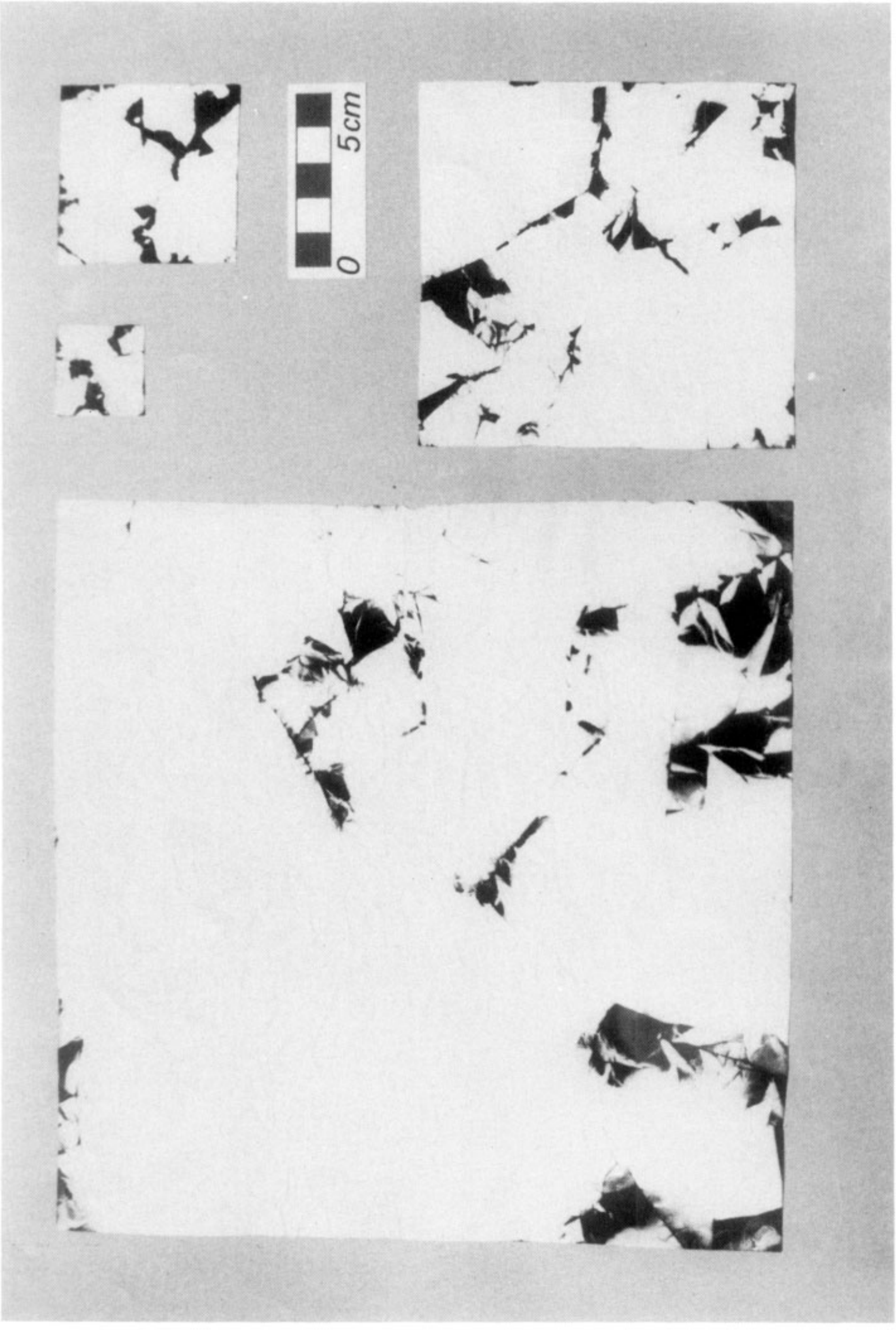

Figure 2. Unfolded balls obtained after air-brush painting with the same pigment as in figure 1.

Thus it is interesting to observe that our experimental result reported in this paragraph may be in agreement with [2] just when the values for d fall in the narrow interval of dimensions expected for a region where turbulent activity is concentrated. Obviously, for the experimental situation described in the second and third paragraphs of this letter d must be identified with the regular value $d = 3$ since in this case the random surfaces are immersed and painted in a fluid which covers completely a tridimensional region.

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